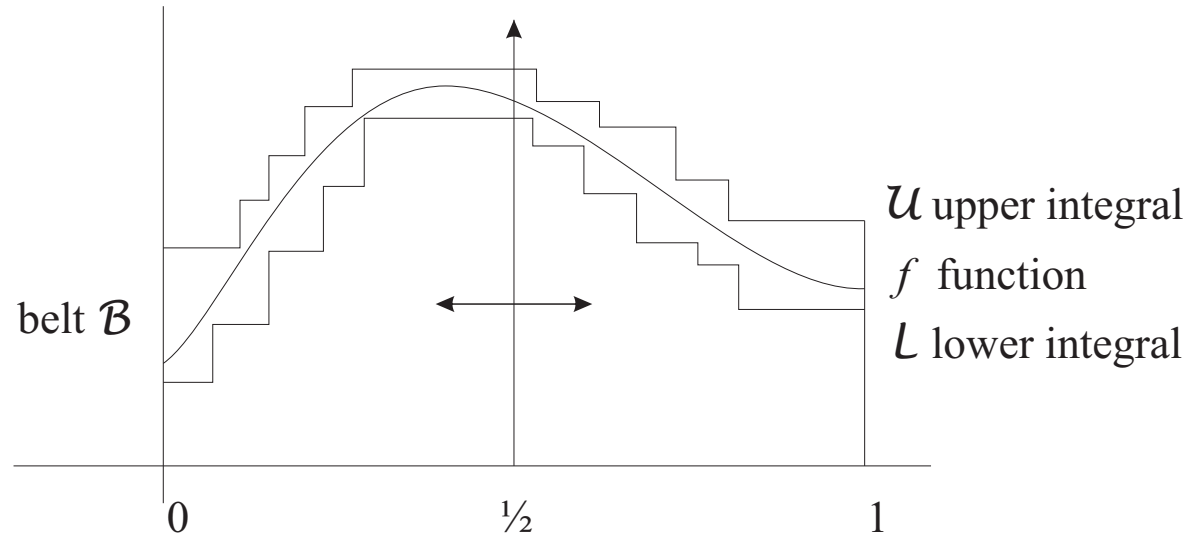


Fundamental Calculus Fact: Continuous function is integrable.

Proof (see [1]).



If on the interval  $I_1=[0,1]$  is the difference  $U-L > \epsilon$  (it means somewhere the “height” of the belt  $\mathcal{B}$  at least  $\epsilon$ ), then after splitting the interval at  $1/2$  into two intervals on one side the difference  $U-L > \epsilon/2$  (and somewhere the “height” of the belt  $\mathcal{B}$  at least  $\epsilon/2$ ). Denote the interval  $I_2$ . We repeat it and in the intersection of the intervals  $I_n$  we find a point with the “height” of the belt  $\mathcal{P}$  at least  $\epsilon/2$ . At this point the function  $f$  cannot be continuous. A contradiction.

[1] Walk, Stephen M.: A Streamlined Proof of an Essential Calculus Fact, American Mathematical Monthly, Volume 117, Number 9, November 2010 , pp. 832-833.