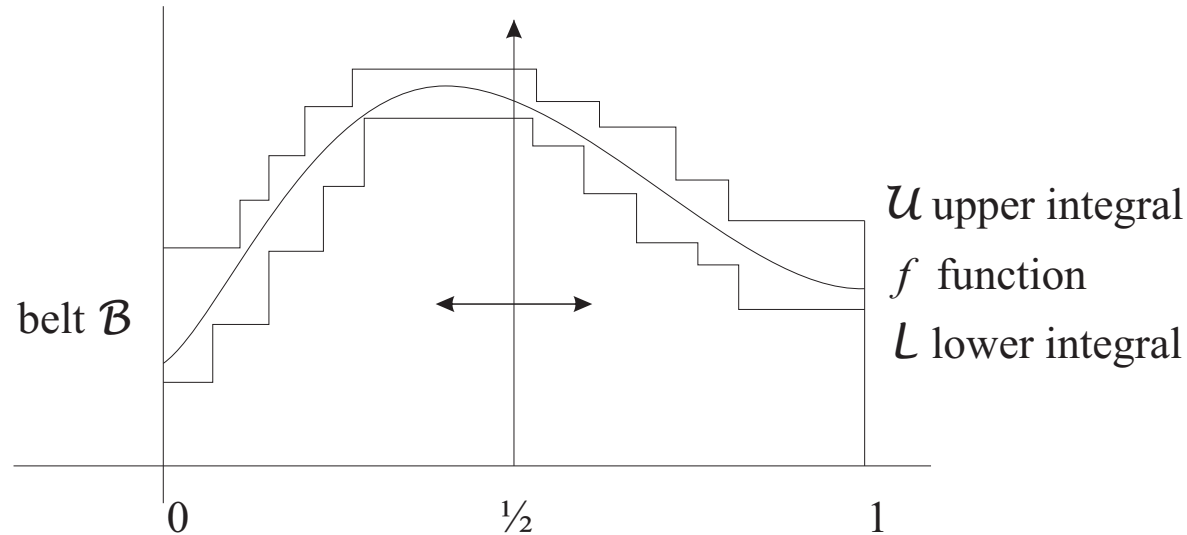


Fundamental Calculus Fact: Continuous function is integrable.

Proof (see [1]).



If on the interval $I_1=[0,1]$ is the difference $U-L > \epsilon$ (it means somewhere the “height” of the belt \mathcal{B} at least ϵ), then after splitting the interval at $1/2$ into two intervals on one side the difference $U-L > \epsilon/2$ (and somewhere the “height” of the belt \mathcal{B} at least $\epsilon/2$). Denote the interval I_2 . We repeat it and in the intersection of the intervals I_n we find a point with the “height” of the belt \mathcal{P} at least $\epsilon/2$. At this point the function f cannot be continuous. A contradiction.

[1] Walk, Stephen M.: A Streamlined Proof of an Essential Calculus Fact, American Mathematical Monthly, Volume 117, Number 9, November 2010 , pp. 832-833.