Posloupnosti funkci

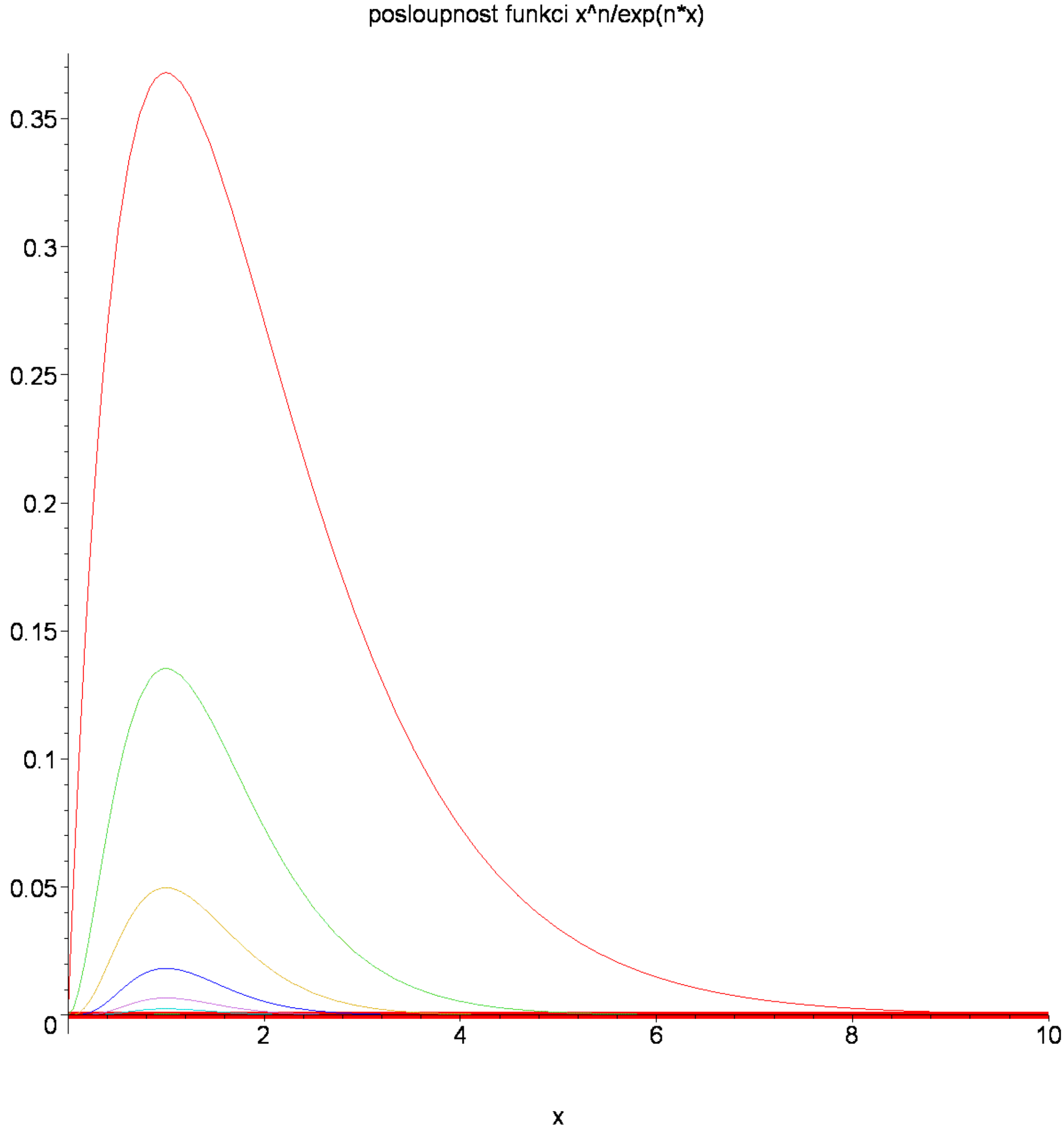
7.a. Vysetrete stejnomernou a bodovou konvergenci posloupnosti funkci

$$f_n(x) := \frac{x^n}{e^{(n\,x)}}$$

```
> p1:=plot({seq(x^n/exp(n*x),n=1..10)},x=0..10):
```

limitni funkce f(x)= $\lim_{n \to \infty} f_n(x)$

```
> f1:=plot({limit(x^n/exp(n*x),n=infinity)},x=0..10,color=red,t
  hickness=8):
> display(p1,f1,title=`posloupnost funkci x^n/exp(n*x)`);
```

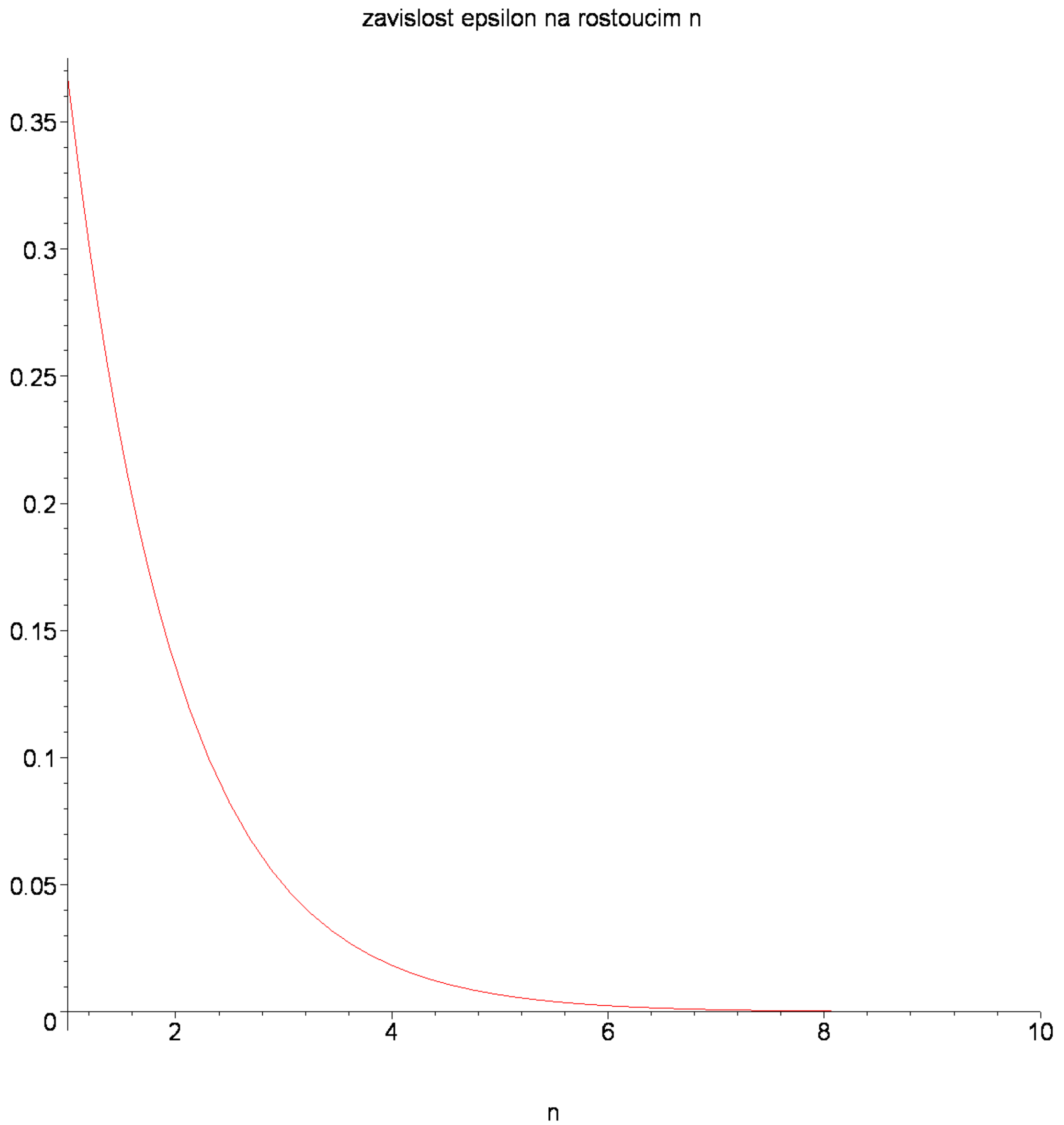


maximum n-te funkce na intervalu <0..10>

```
> solve(diff(x^n/exp(n*x),x)=0,x);
```

$$1$$

```
> plot(1/exp(n),n=1..10,title=`zavislost epsilon na rostoucim
  n`);
```




```
> limit(1/exp(n),n=infinity);
```

$$0$$

Co z toho vseho usoudit, opravdu nevim, udelam si nejaky vzorovy priklad :)

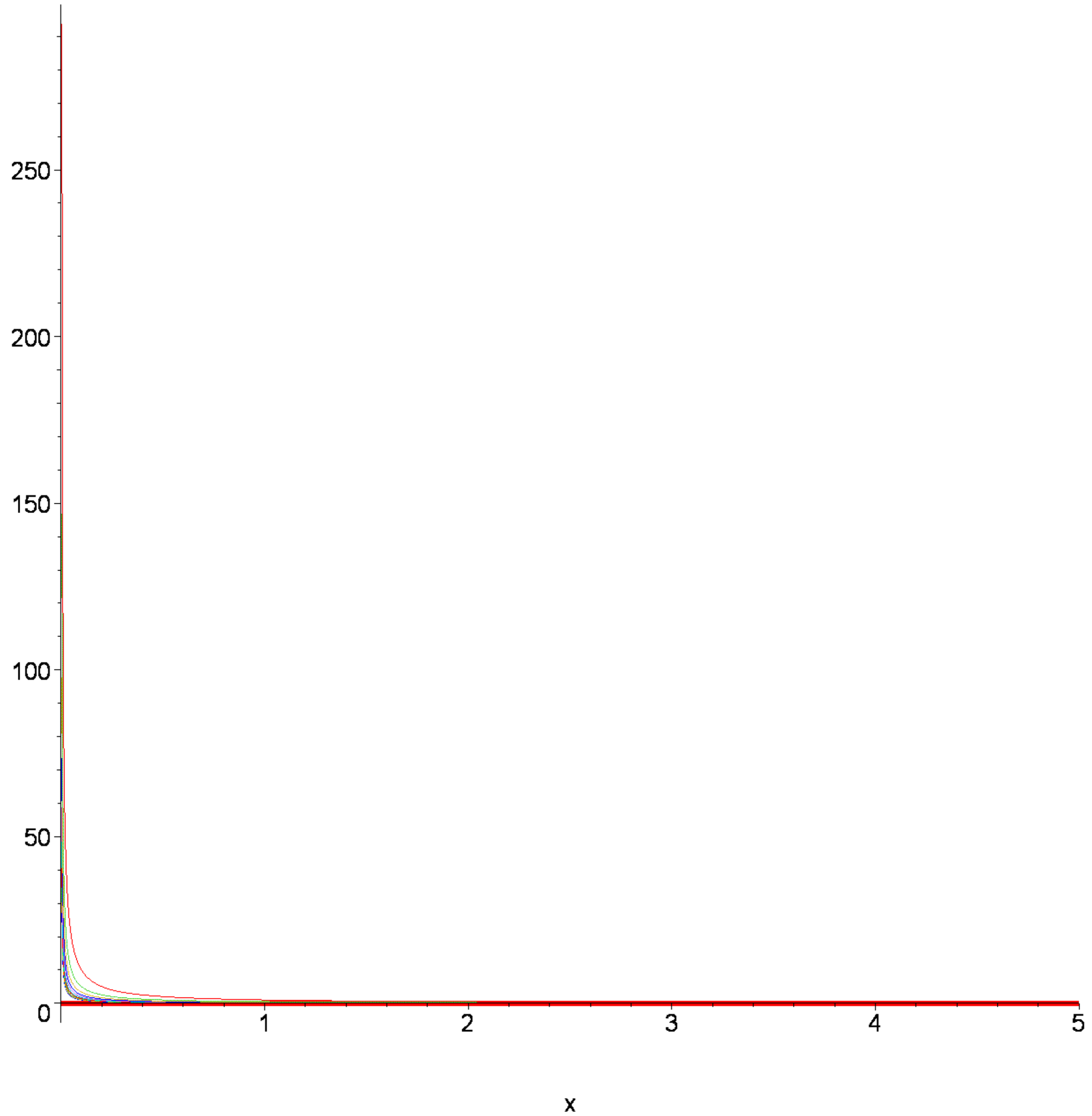
7.b.Vysetrete bodovou a stejnomernou konvergenci rady

$$f_n(x) = \frac{1}{n\,x}$$

```
> p3:=plot({seq(1/(n*x),n=1..10)},x=0..5):
```

limitni funkce f(x)= $\lim_{n \to \infty} f_n(x)$

```
> f3:=plot({limit(1/(n*x),n=infinity)},x=0..5,color=red,thickne
  ss=6):
> display(p3,f3);
```



Jiz z obrazku je zrejme, ze na intervalu ? < δ, ∞) funkce stejomerne konverguji k 0, overime to.

$\sigma_n = sup \left| f_n(x) - \mathrm{f}(x) \right|$

se rovna funkci $f_n(x)$a to pro n -> ∞ konverguje k nule. Takze nase posloupnot stejnomerne konverguje na intervalu ? < δ, ∞)

8.Vysetrete stejnomernou a bodovou konvergenci rady

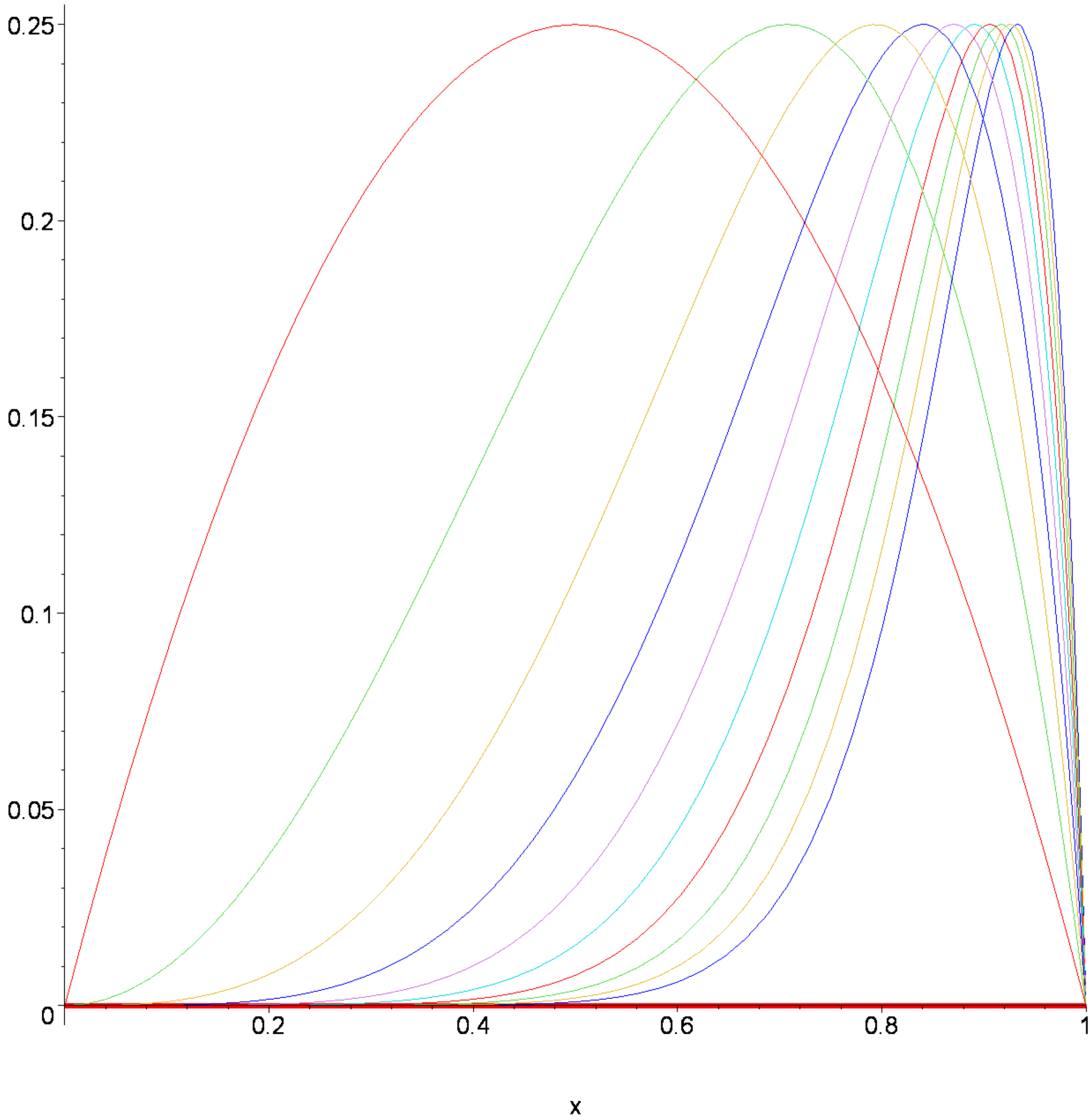
$f_n(x) = x^n - x^{(2\,n)}$

```
> p2:=plot({seq(x^n-x^(2*n),n=1..10)},x=0..1):
```

limitni funkce f(x)= $\lim_{n \to \infty} f_n(x)$

```
> f2:=plot({limit(x^n-x^(2*n),n=infinity)},x=0..1,color=red,thi
  ckness=6):
```

```
> display(p2,f2);
```



Vidime, ze funkce bodove konverguje k 1. Nyni budeme vysetrovat stejnomernou konvergnci.

Z prubehu funce zjistime $\sigma_n = sup \left| f_n(x) - \mathrm{f}(x) \right|$

```
> g:=x^n-x^(2*n);
```

$$g := x^n - x^{(2\,n)}$$

```
> h:=diff(g,x);
```

$$h := \frac{x^n\, n}{x} - \frac{2\, x^{(2\,n)}\, n}{x}$$

```
> h:=unapply(h,x);
```

$$h := x \rightarrow \frac{x^n\, n}{x} - \frac{2\, x^{(2\,n)}\, n}{x}$$

```
> solve(h(x)=0,x);
```

$$\mathbf{e}^{\left(-\frac{\ln(2)}{n}\right)}$$

Mappl jeste opomenul, ze rovnice ma reseni pro x=0, ale nas stejne bude zajimat toto reseni. Dosadime ho do g(x) a pokud bude vysledek pro n-> ∞ konvergovat k nule, funkce budou stejnomerne konvergentni.

```
> g:=x->x^n-x^(2*n);
```

$$g := x \rightarrow x^n - x^{(2\,n)}$$

```
> d:=g(exp(-ln(2)/n));
```

$$d := \left(\mathbf{e}^{\left(-\frac{\ln(2)}{n}\right)}\right)^n - \left(\mathbf{e}^{\left(-\frac{\ln(2)}{n}\right)}\right)^{(2\,n)}$$

```
> limit(d,n=infinity);
```

$$\frac{1}{4}$$

..coz je nase hledane σ_n

Rada stejnomerne nekonverguje na intervalu <0,1>

```
>
```