```
>
```

Spočteme $\lim_{x \to \infty} \arctan(x)$.

V Maple použijeme příkaz **limit** pro funkci $\arctan(x)$.

```
> Limit(arctan(x), x=infinity)=limit(arctan(x), x=infinity);
```

$$\lim_{x \to \infty} \arctan(x) = \frac{\pi}{2}$$

```
> evalf(%);
```

$$1.570796327 = 1.570796327$$

```
>
```

Připomeneme si "epsilon" definici limity funkce

**Definice:** Nechť je $f(x)$ definovaná pro všechna $x$ větší než dané $x_0$.
Pak $f(x)$ má limitu A pro $x$ konvergující k $\infty$, píšeme

$$\lim_{x \to \infty} f(x) = A,$$

jestliže pro každé (malé) kladné $\varepsilon$ existuje číslo $K$ tak, že $|f(x) - A| < \varepsilon$ pro všechna $x > K$.

Pro malé $\varepsilon$ najdeme číslo $K$ tak že

$$\frac{\pi}{2} - \varepsilon < \arctan(x) < \frac{\pi}{2} + \varepsilon$$

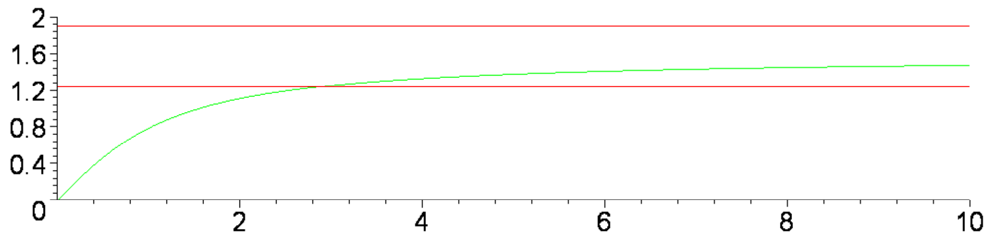
pro všechna $x > K$, pak

$$\lim_{x \to \infty} \arctan(x) = \frac{\pi}{2}.$$

Pro $\varepsilon = \frac{1}{3}$ se podíváme na obrázek (zobrazíme i konstantní funkce $\frac{\pi}{2} - \varepsilon$ a $\frac{\pi}{2} + \varepsilon$ ).

$$x_0$$

```
> eps:=1/3:
> plot([Pi/2-eps,arctan,Pi/2+eps], 0..10, 0..2,
  color=[red,green,red], scaling = constrained);
```




```
> solve(abs(arctan(x)-Pi/2)<eps, x);
```

$$\mathrm{RealRange}\left(\mathrm{Open}\left(\cot\left(\frac{1}{3}\right)\right), \infty\right)$$

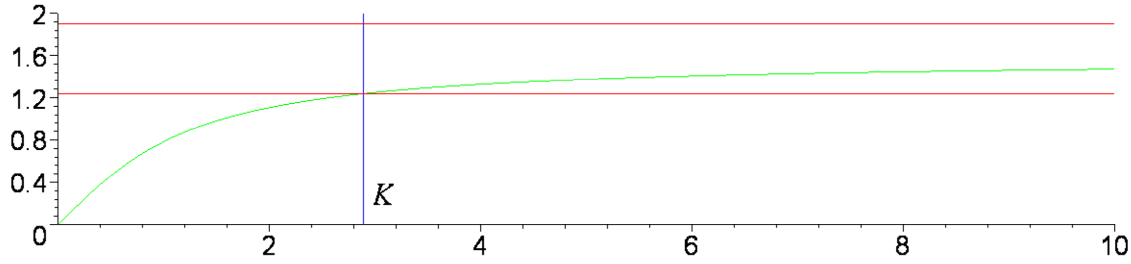
```
> K:=evalf(cot(eps));
```

$$K := 2.888057037$$

To je hledane *K*.

```
> plota:=plot([Pi/2-eps,arctan,Pi/2+eps], 0..10, 0..2, 
  color=[red,green,red], scaling = constrained):

> plotb:=plot([K, t, t=0..2.5],color=blue):
> plotc := plots[textplot]([K+0.1, 0.3, "K"], 
  align=RIGHT,font=[TIMES,ITALIC,12]):
> display(plota,plotb, plotc);
```




```
>
> f := x -> (sin(x)+2*exp(x))/(cos(x)+exp(x));
```

$$f := x \rightarrow \frac{\sin(x) + 2\,\mathbf{e}^x}{\cos(x) + \mathbf{e}^x}$$

```
> limit(f(x), x=infinity);
```

$$\lim_{x \rightarrow \infty} \frac{\sin(x) + 2\,\mathbf{e}^x}{\cos(x) + \mathbf{e}^x}$$

```
> g := x -> (sin(x)*exp(-x)+2)/(cos(x)*exp(-x)+1);
```

$$g := x \rightarrow \frac{\sin(x)\,\mathbf{e}^{(-x)} + 2}{\cos(x)\,\mathbf{e}^{(-x)} + 1}$$

```
> limit(g(x), x=infinity);
```

$$2$$

```
> simplify (f(x)-g(x));
```

$$0$$

```
>
>
>
>
```

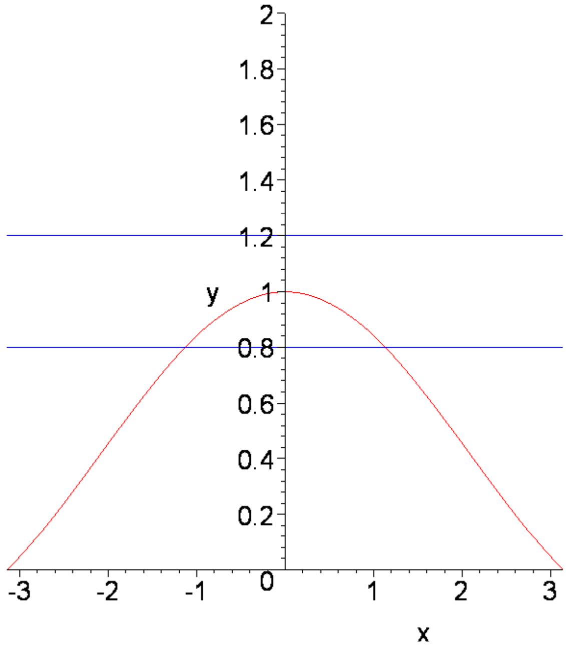
Zkusíme

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x},$$

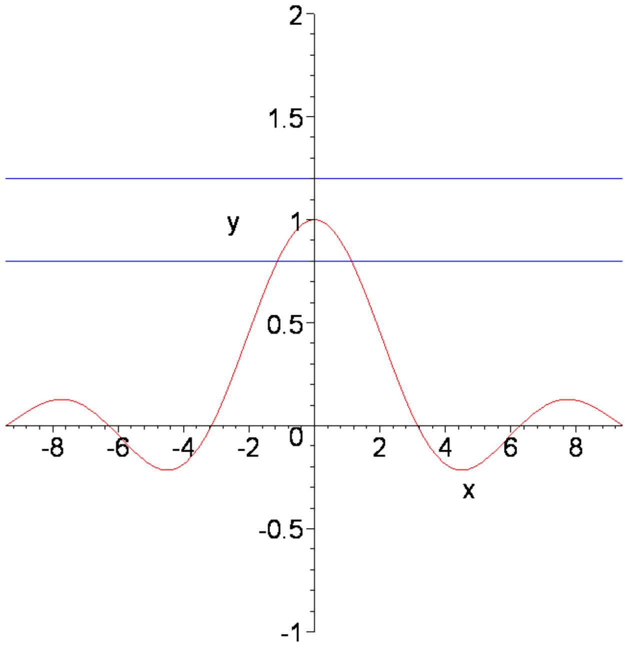
```
> Limit(sin(x)/(x),x=0)=limit(sin(x)/(x),x=0);
```

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

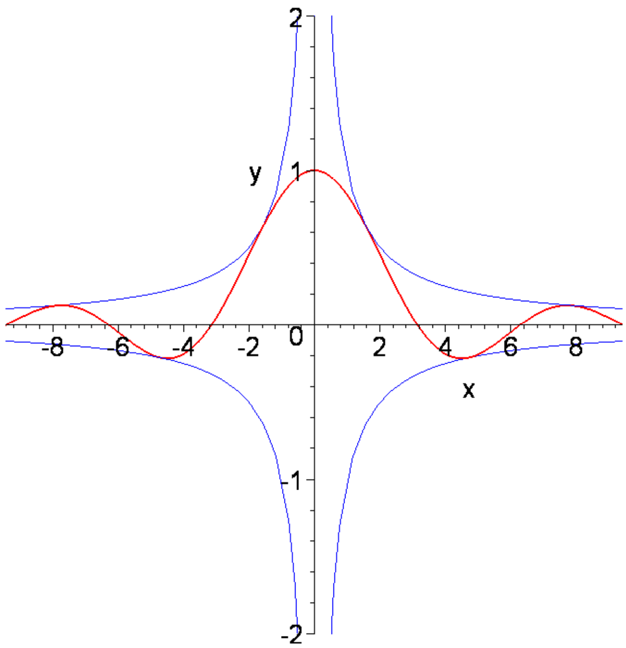
```
> plot([0.8,sin(x)/(x),1.2], x=-Pi..Pi, y=0..2,
  color=[BLUE,red,BLUE]);
```



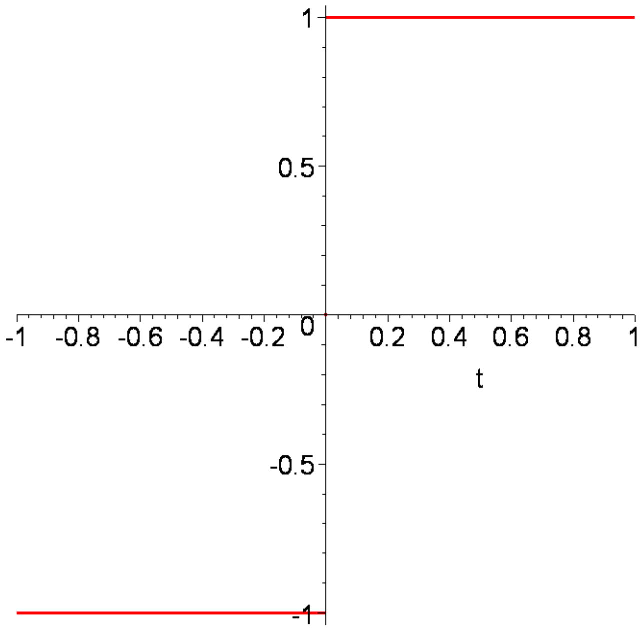

```
> plot([0.8,sin(x)/(x),1.2], x=-3*Pi..3*Pi, y=-1..2,
  color=[BLUE,red,BLUE]);
```




```
> plot([1/x,sin(x)/(x),-1/x], x=-3*Pi..3*Pi, y=-2..2,
  color=[BLUE,red,BLUE], thickness=[1,2,1]);
```




```
>
>
> plot(signum(t), t=-1..1, color=red, thickness=3, discont=true);
```




```
> Limit(signum(x), x=0, right) = limit(signum(x), x=0, right);
```

$$\lim_{x \to 0+} \operatorname{signum}(x) = 1$$

```
> Limit(signum(x), x=0, left) = limit(signum(x), x=0, left);
```

$$\lim_{x \to 0-} \operatorname{signum}(x) = -1$$

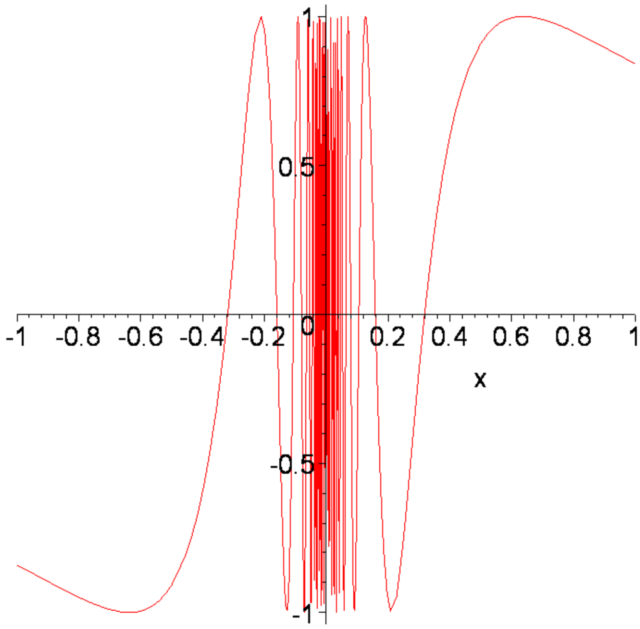
```
> Limit(signum(x), x=0) = limit(signum(x), x=0);
```

$$\lim_{x \to 0} \operatorname{signum}(x) = \mathit{undefined}$$

```
>
>
> f := x -> sin(1/x);
```

$$f := x \to \sin\left(\frac{1}{x}\right)$$

```
> plot(f(x), x=-1..1);
```




```
> limit(f(x), x=0);
```

$$-1\,..\,1$$

```
>
>
>
```