```
> restart:
> plotsetup(default, plotoptions=`axiswidth=1in, axisheight=1in`);
```

Zvolte funkci f a zobrazte funkci g = f ' spolu s funkcí

$$\frac{f(x+h)-f(x)}{h}$$

pro ruzné hodnoty h. Overte, že

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = g(x)$$

Procedura funguje napriklad pro
f(x)=sin(x), a=0, b=2*Pi, c=-1, d=1
f(x)=x^2, a=0, b=2, c=0, d=4
f(x)=arctan(x), a=-2, b=2, c=-2, d=2

```
> f:=x->sin(x);
```

$$f := x \to \sin(x)$$

```
> a:=0;b:=2*Pi;
```

$$a := 0$$
$$b := 2\,\pi$$

```
> c:=-1;d:=1;
```

$$c := -1$$
$$d := 1$$

```
> g:=diff(f(x),x);
```

$$g := \cos(x)$$

Jak funguje prikaz Limit a limit vidime zde (prikaz lim nefunguje):

```
> Limit((f(x+h)-f(x))/h,h=0) = limit((f(x+h)-f(x))/h,h=0);
```

$$\lim_{h \to 0} \frac{\sin(x+h)-\sin(x)}{h} = \cos(x)$$

```
>
```

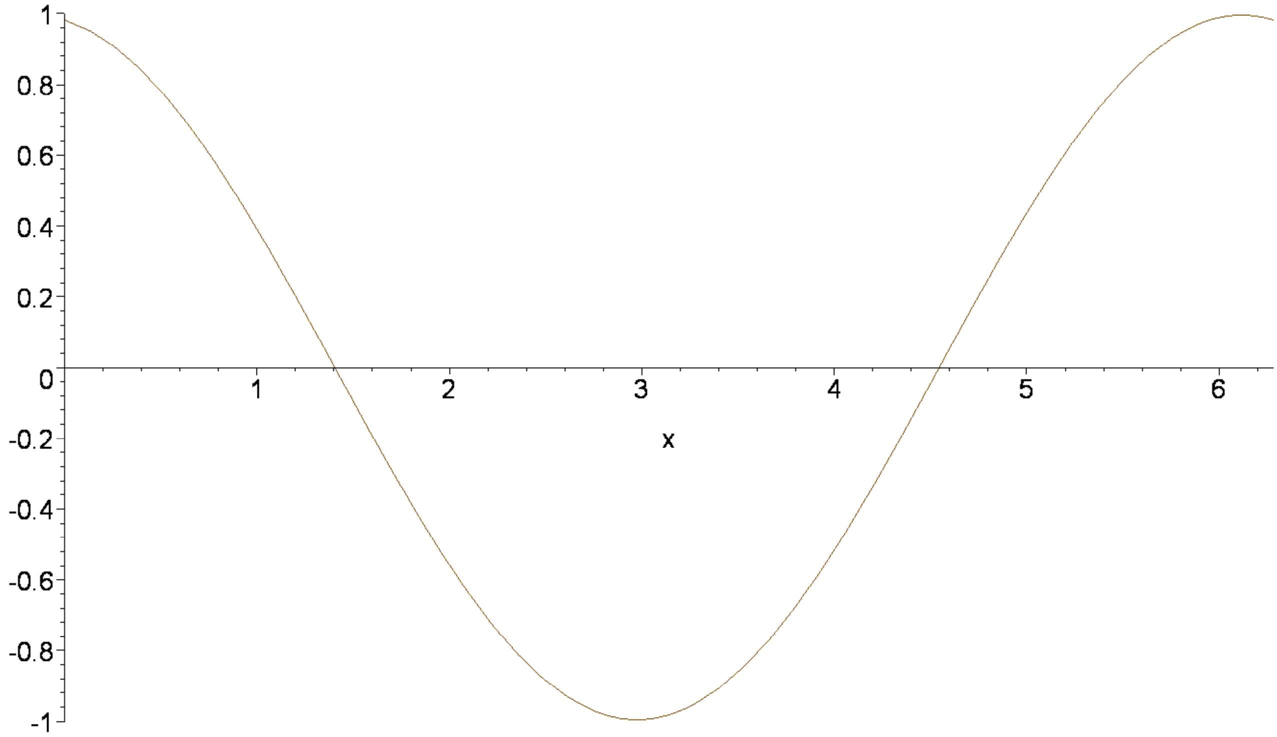
Musime nacist knihovny pro ploty:

```
> with(plots):
>
> C:=plot(g(x),x=a..b, color=red, thickness=3):
>
```

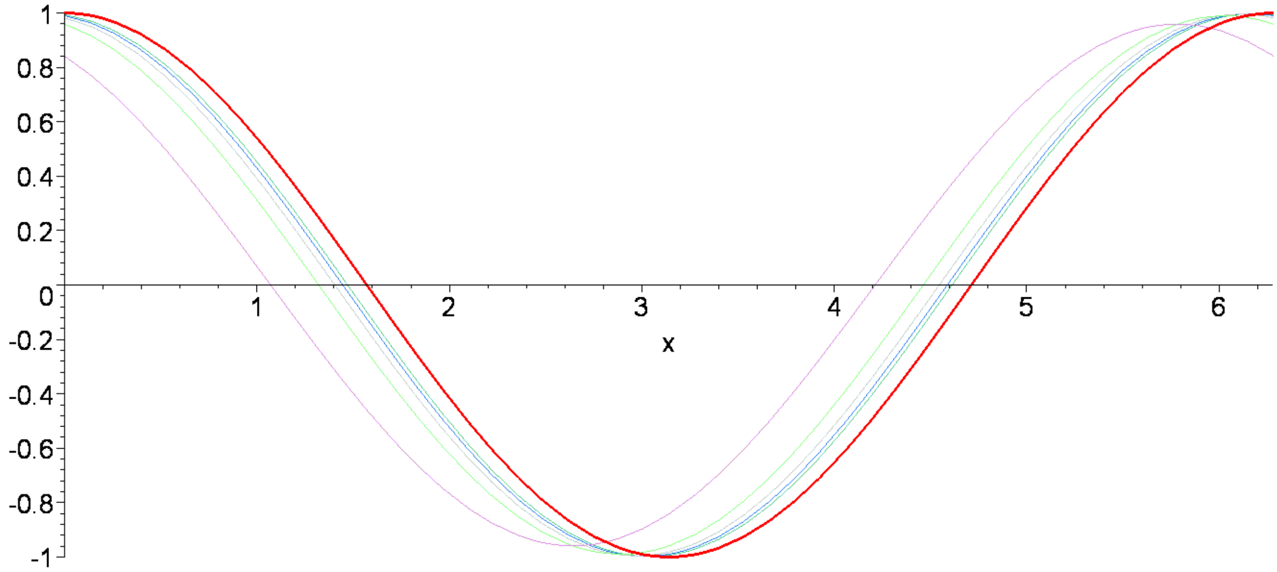
pouzijeme nahodne barvicky:

```
> P:=n->plot((f(x+1/n)-f(x))/(1/n),x=a..b,
  c..d,color=COLOR(RGB,rand()/10^12,rand()/10^12,rand()/10^12)):
```

```
> P(3);
```

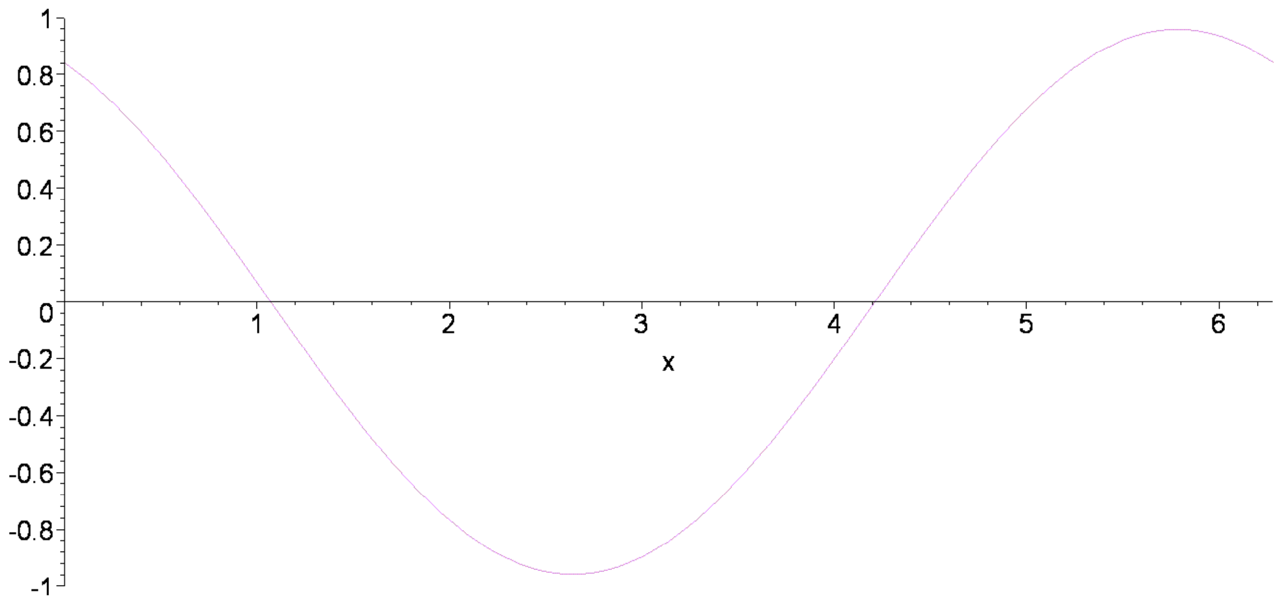



```
> L:=seq(P(n), n=1..5):
> display(C,L, insequence=false);
```



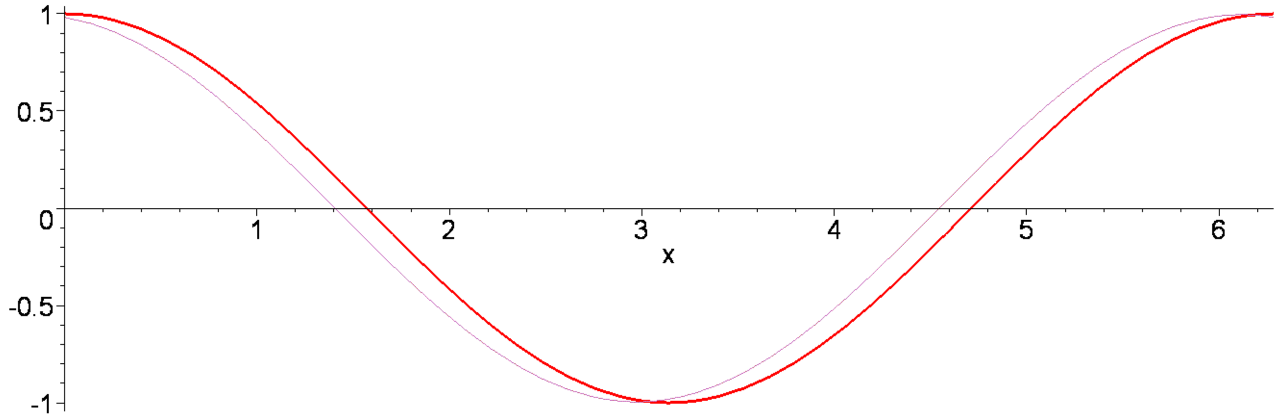
TADY PUJDE O ANIMACI, KLIKNEME NA VYTVORENY PLOT A POUZIVAME OVLADATKA (PLAY a pod) v horni liste:

```
> display(L,C, insequence=true);
```

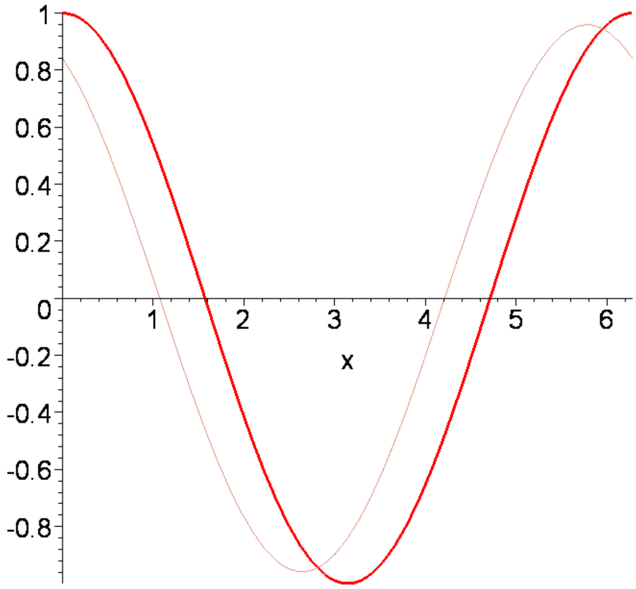


tady udelame dvojice plotu (aproximace s nahodnou barvou a limita tluste cervene):

```
>
> PC:=n->plot({(f(x+1/n)-f(x))/(1/n),g(x)},x=a..b,
  color=[COLOR(RGB,rand()/10^12,rand()/10^12,rand()/10^12),
  red], thickness=[1,3]):
> PC(3);
```




```
> LPC:=seq(PC(n), n=1..5):
> display(LPC, insequence=true);
```



A ZMACKNEME NA GRAF A TLACITKO PLAY :-)

A muzeme animaci zpomalit (nahore na liste jako u prehravace)

```
>
>
>
>
> with(plots):
> f := x -> x; g := x -> 1/x;
```

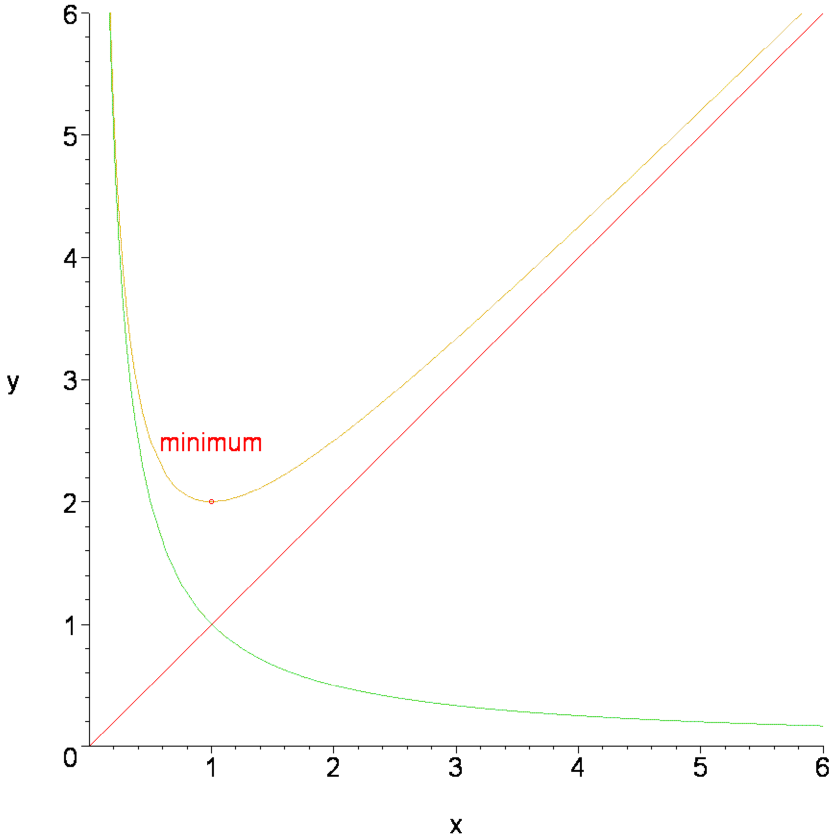
$$f := x \rightarrow x$$

$$g := x \rightarrow \frac{1}{x}$$

```
> h := x -> x + 1/x;
```

$$h := x \rightarrow x + \frac{1}{x}$$

```
> plot1 := plot({f(x), g(x), h(x)}, x=0..6, y=0..6):
> plot2 := plots[textplot]({[1,2.5,`minimum`]}):
> Puntik := [[1,2]]:
> plot3 := plot(Puntik, style=POINT, symbol=CIRCLE):
>
> plots[display]({plot1, plot2, plot3}, scaling=constrained);
```



>

Polynom 10 stupně

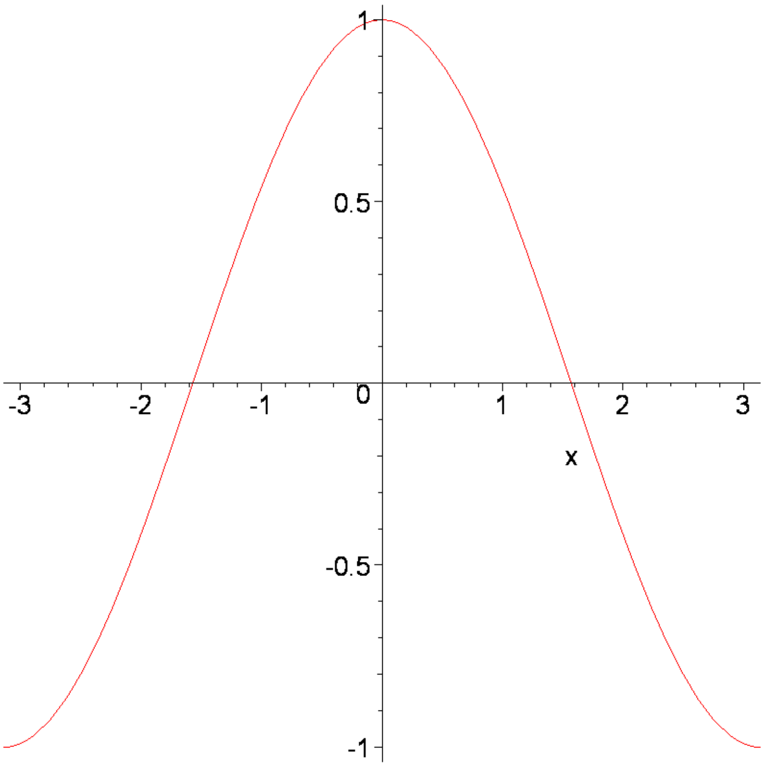
$$\sum_{m=0}^{5} \frac{(-1)^m x^{(2m)}}{(2m)!} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \ldots - \frac{x^{10}}{10!} .$$

zadáme Maple V příkazem **sum** :

```
> S := x -> sum((-1)^m*x^(2*m)/((2*m)!), m=0..5);
```

$$S := x \rightarrow \sum_{m=0}^{5} \frac{(-1)^m x^{(2m)}}{(2m)!}$$

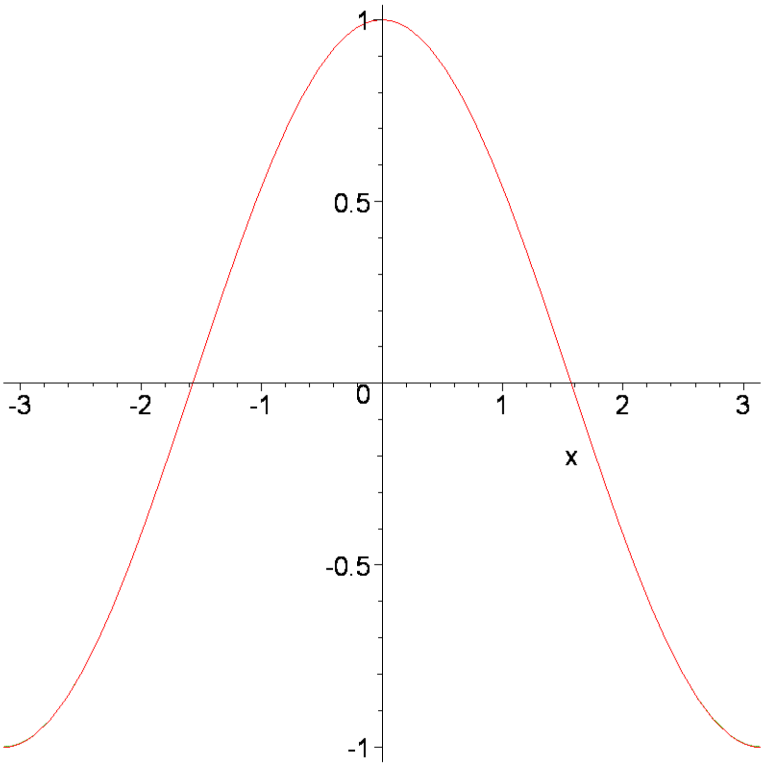
```
> plot(S(x), x=-Pi..Pi);
```



>

...aha, to asi bude cos(x) , zkusíme to nakreslit společně ...

```
> plot({cos(x),S(x)}, x=-Pi..Pi);
```



>

O.K. :-)

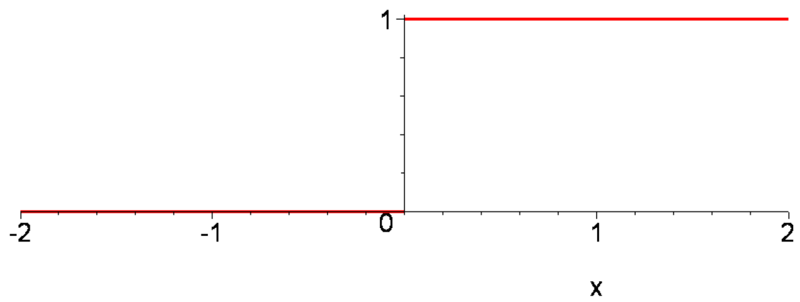
>

>

Heaviside - ova funkce H(x)

$$H(x) = 1 \text{ pro } x > 0,$$
$$H(x) = 0 \text{ pro } x < 0.$$

```
> plot(Heaviside(x), x=-2..2, scaling=constrained, ytickmarks=2,
  thickness=3, discont=true);
```




```
> U := t -> piecewise(t<0, 0, t>0, 1);
```

$$U := t \rightarrow \text{piecewise}(t < 0, 0, 0 < t, 1)$$

>

Maple V pomocí příkazu **convert** pozná, o kterou funkci jde. Zde převede po částech definovanou funkci **abs(x)** do ekvivalentního tvaru pomocí Heaviside-ovy funkce a do definice po castech:

```
> convert(abs(x), Heaviside);
```

$$-x + 2\,x\,\text{Heaviside}(x)$$

```
> convert(abs(x), piecewise);
```

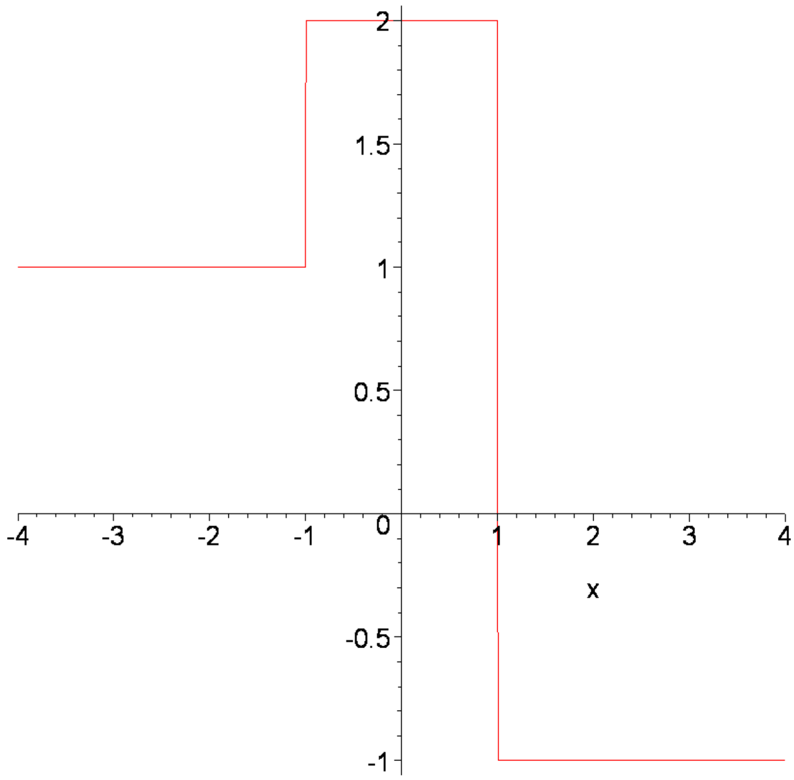
$$\begin{cases} -x & x < 0 \\ x & 0 \le x \end{cases}$$

>

```
> f := x -> piecewise( x<-1, 1, x>=-1 and x<1, 2, x>=1, -1);
```

$$f := x \rightarrow \text{piecewise}(x < -1, 1, -1 \le x \textbf{ and } x < 1, 2, 1 \le x, -1)$$

```
> plot(f(x), x=-4..4);
```




```
> 
> convert(f(x), Heaviside, x);
```

$$1+\text{Heaviside}(x+1)-3\,\text{Heaviside}(-1+x)$$

```
> convert(f(x), piecewise, x);
```

$$\begin{cases} 1 & x<-1 \\ 2 & x<1 \\ -1 & 1\le x \end{cases}$$

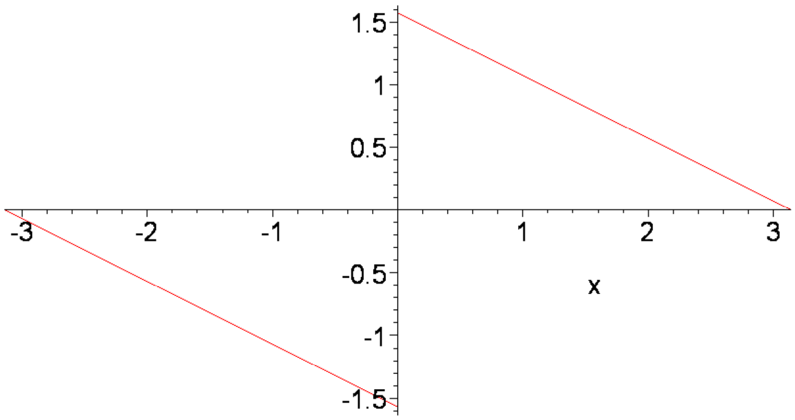
```
> 
> 
> f := x -> piecewise( x<0, -(x+Pi)/2, x>=0, (Pi-x)/2);
```

$$f := x \to \text{piecewise}\left(x<0, -\frac{1}{2}x-\frac{\pi}{2}, 0\le x, \frac{\pi}{2}-\frac{1}{2}x\right)$$

```
> convert(f(x), piecewise, x);
```

$$\begin{cases} -\frac{x}{2}-\frac{\pi}{2} & x<0 \\ \frac{\pi}{2}-\frac{x}{2} & 0\le x \end{cases}$$

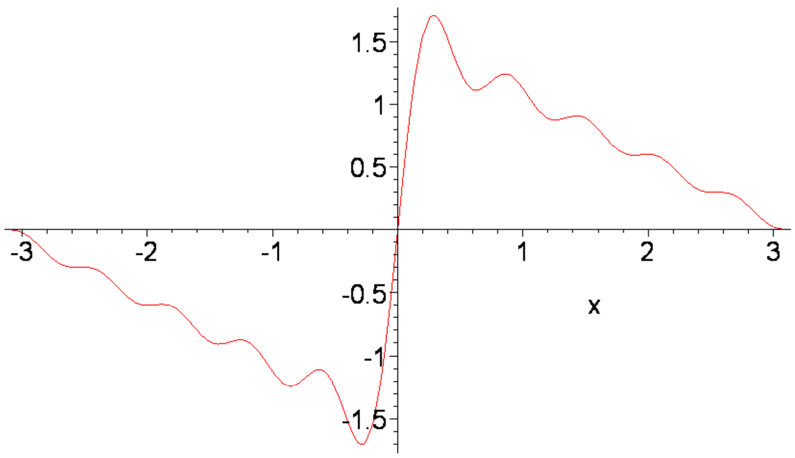
```
> plot(f(x), x=-Pi..Pi, scaling=constrained);
```




```
> k:=10:
> g:= x->sum(sin(n*x)/n, n=1..k);
```

$$g := x \to \sum_{n=1}^{k} \frac{\sin(n\,x)}{n}$$

```
> plot(g(x), x=-Pi..Pi, scaling=constrained);
```




```
>
>
```

>

Nalezněte největší kladné α takové, že $|x-2| < \alpha$ implikuje $|x^3-8| < 1$.
Vidíme, že pro $x = 2$ je $|x^3-8| = 0$, a tak hledáme asi malá α.
Ale pro $x = 3$ je $|x-2| = 1$ a tedy $|x^3-8| = 21 > 1$.
Tedy je $\alpha < 1$.
Sestrojíme graf

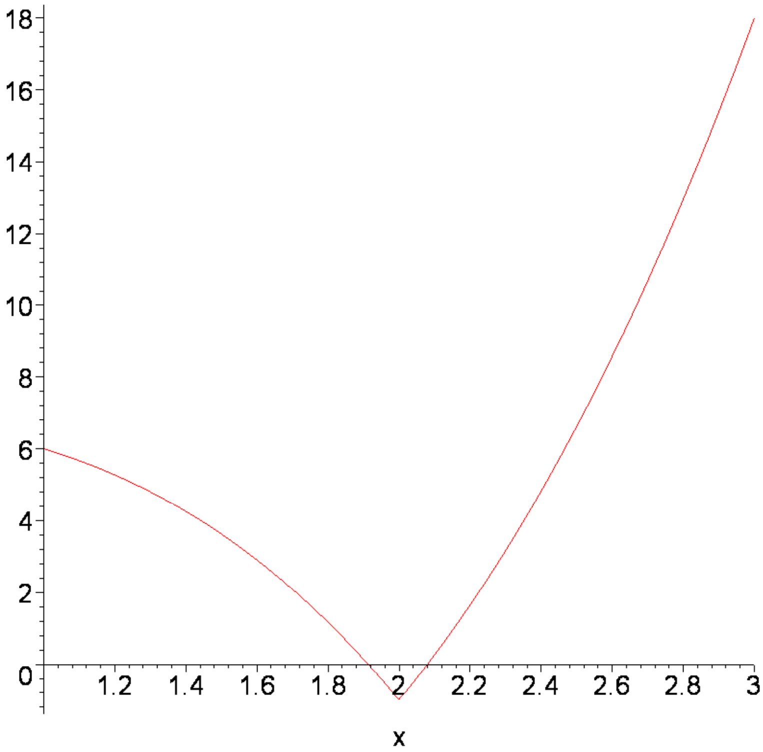
$$f(x) = |x^3-8| - 1$$

na intervalu $[1, 3]$.

```
> f:= x -> abs(x^3-8)-1;
```

$$f := x \to |x^3-8| - 1$$

```
> plot(f(x), x=1..3);
```



>

Tedy f(x) je záporná na maličkém intervalu okolo $x = 2$.
Pomocí myši zjistíme interval ~ (1.9, 2.08).
Tedy α bude asi menší než 2 – 1.9 a 2.08 – 2, tedy asi .8e-1
Jaké je přesně α ?

```
> solve(f(x)<0, x);
```

$$\mathrm{RealRange}(\mathrm{Open}(7^{(1/3)}), \mathrm{Open}(3^{(2/3)}))$$

>

Na intervalu $(7^{\left(\frac{1}{3}\right)}, 3^{\left(\frac{2}{3}\right)})$ je $f(x) = |x^3-8| - 1 < 0$.

Tedy α musí být menší než čísla $\left|7^{\left(\frac{1}{3}\right)} - 2\right|$ a $\left|3^{\left(\frac{2}{3}\right)} - 2\right|$. Pomocí příkazu **min** dostaneme

```
> alpha := min(abs(7^(1/3)-2), abs(3^(2/3)-2));
```

$$\alpha := 3^{(2/3)} - 2$$

```
> evalf(alpha);
```

$$0.080083823$$

```
>
```

Tedy největší α takové, že $|x - 2| < \alpha$ implikuje $|x^3 - 8| < 1$ je $\alpha = 3^{\left(\frac{2}{3}\right)} - 2$, přibližně .80083823e-1.
To jsme na grafu uviděli rychleji ...

```
>
>
>
>
>
>
```