```
> restart:
```

Použití derivací k sestrojení grafu

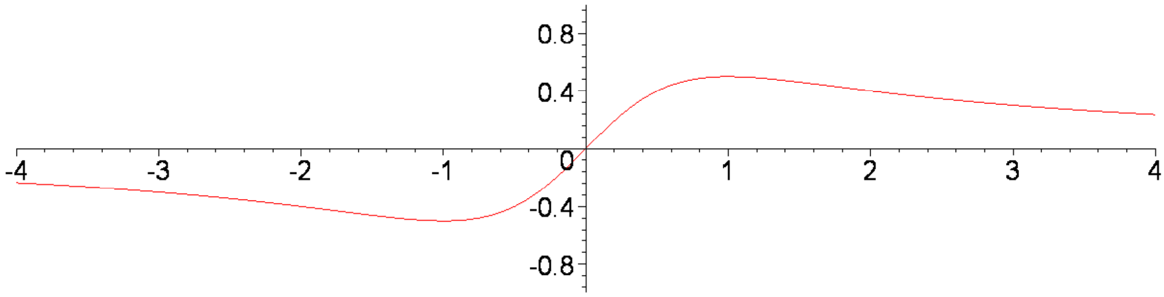
Ukážeme si, jak na tyto body přijít ...

- má-li funkce derivaci a lokální extrém, je derivace nulová
- má-li funkce druhou derivaci a bof inflexe, je druhá derivace nulová
- v kritickém bodě rozhoduje monotonie první derivace (nebo druhá derivace)

```
> f := x-> x/(x^2+1);
```

$$f := x \to \frac{x}{x^2+1}$$

```
> a:=-4:b:=4:c:=-1:d:=1:
>
>
> plot(f,a..b, c..d , scaling=constrained);
```



```
> dfdx := D(f);
```

$$dfdx := x \rightarrow \frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2}$$

```
> bodiky := solve(dfdx(x)=0, x);
```

$$bodiky := -1, 1$$

```
>
> d2fdx2 := D(dfdx);
```

$$d2fdx2 := x \rightarrow -\frac{6\,x}{(x^2+1)^2} + \frac{8\,x^3}{(x^2+1)^3}$$

```
>
> map(d2fdx2, [bodiky]);
```

$$\left[\frac{1}{2}, \frac{-1}{2}\right]$$

```
> map(f, [bodiky]);
```

$$\left[\frac{-1}{2}, \frac{1}{2}\right]$$

```
>
> CritPoints := map(z -> fsolve(dfdx(x)=0, x, z), [.6..1, 1..1.6, 
  3..3.6, 4.5..5]);
```

$$CritPoints := \left[1., 1., \mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 3 .. 3.6\right), \mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 4.5 .. 5\right)\right]$$

```
>
> d2fdx2:=D(dfdx);
```

$$d2fdx2 := x \rightarrow -\frac{6\,x}{(x^2+1)^2} + \frac{8\,x^3}{(x^2+1)^3}$$

```
> map(d2fdx2, CritPoints);
```

$$\left[-0.500000000, -0.500000000, -\frac{6\,\mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 3 .. 3.6\right)}{\left(\mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 3 .. 3.6\right)^2 + 1\right)^2} + \frac{8\,\mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 3 .. 3.6\right)^3}{\left(\mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 3 .. 3.6\right)^2 + 1\right)^3}, -\frac{6\,\mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 4.5 .. 5\right)}{\left(\mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 4.5 .. 5\right)^2 + 1\right)^2} + \frac{8\,\mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 4.5 .. 5\right)^3}{\left(\mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 4.5 .. 5\right)^2 + 1\right)^3}\right]$$

```
> map(f, CritPoints);
```

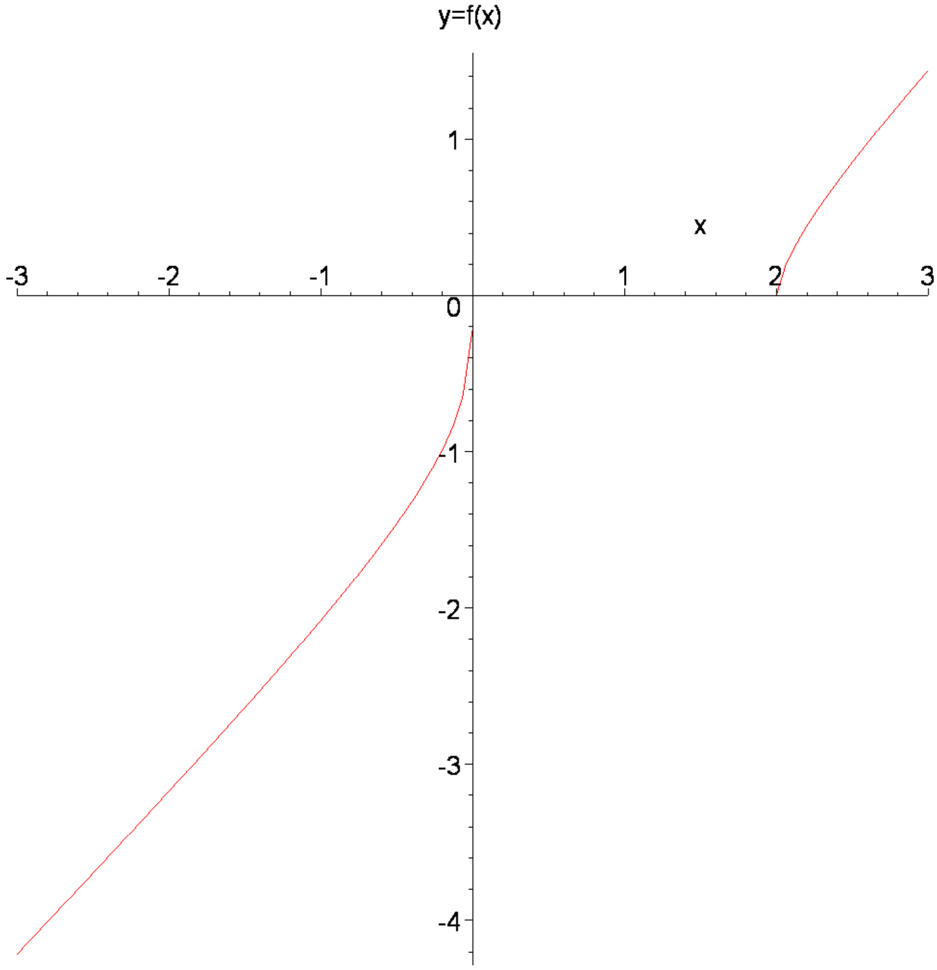
$$\left[0.5000000000, 0.5000000000, \frac{\mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 3 .. 3.6\right)}{\mathrm{fsolve}\left(\frac{1}{x^2+1} - \frac{2\,x^2}{(x^2+1)^2} = 0, x, 3 .. 3.6\right)^2 + 1},\right.$$

$$\frac{\text{fsolve}\left(\frac{1}{x^2+1}-\frac{2\,x^2}{(x^2+1)^2}=0,\,x,\,4.5\,..\,5\right)}{\text{fsolve}\left(\frac{1}{x^2+1}-\frac{2\,x^2}{(x^2+1)^2}=0,\,x,\,4.5\,..\,5\right)^2+1}\,]$$

```
>
>
> f := x -> x^(1/3)*(x-2)^(2/3);
```

$$f := x \to x^{(1/3)}\,(x-2)^{(2/3)}$$

```
> plot(f(x), x=-3..3, title=`y=f(x)`);
```



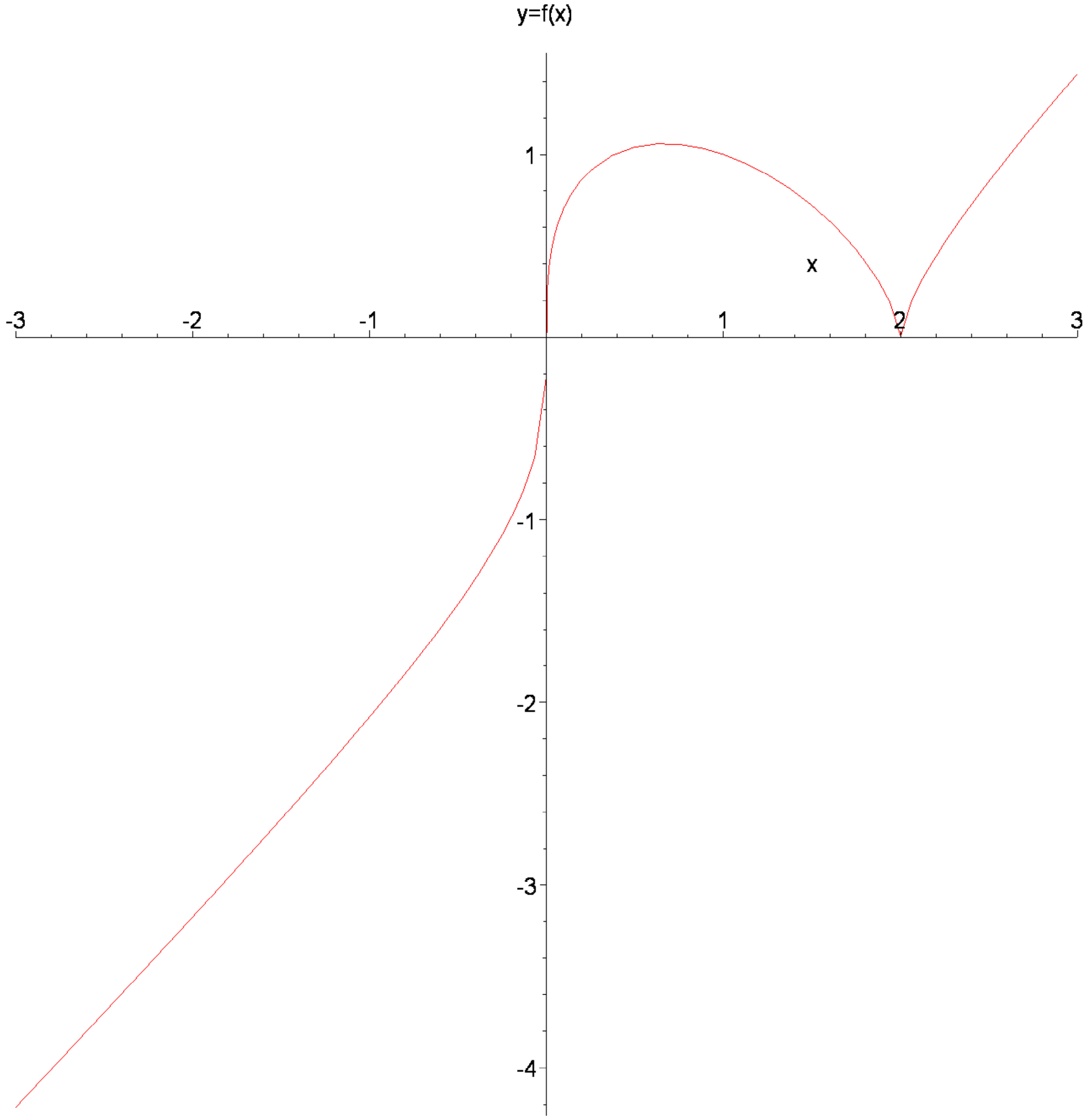

```
>
```

Maple V nic nepoví o funkci pro $x \le 2$. To je tím, jak Maple V zachází s komplexními čísly. Potíže odstraníme pomocí příkazu **surd** (použití je snadné) :.

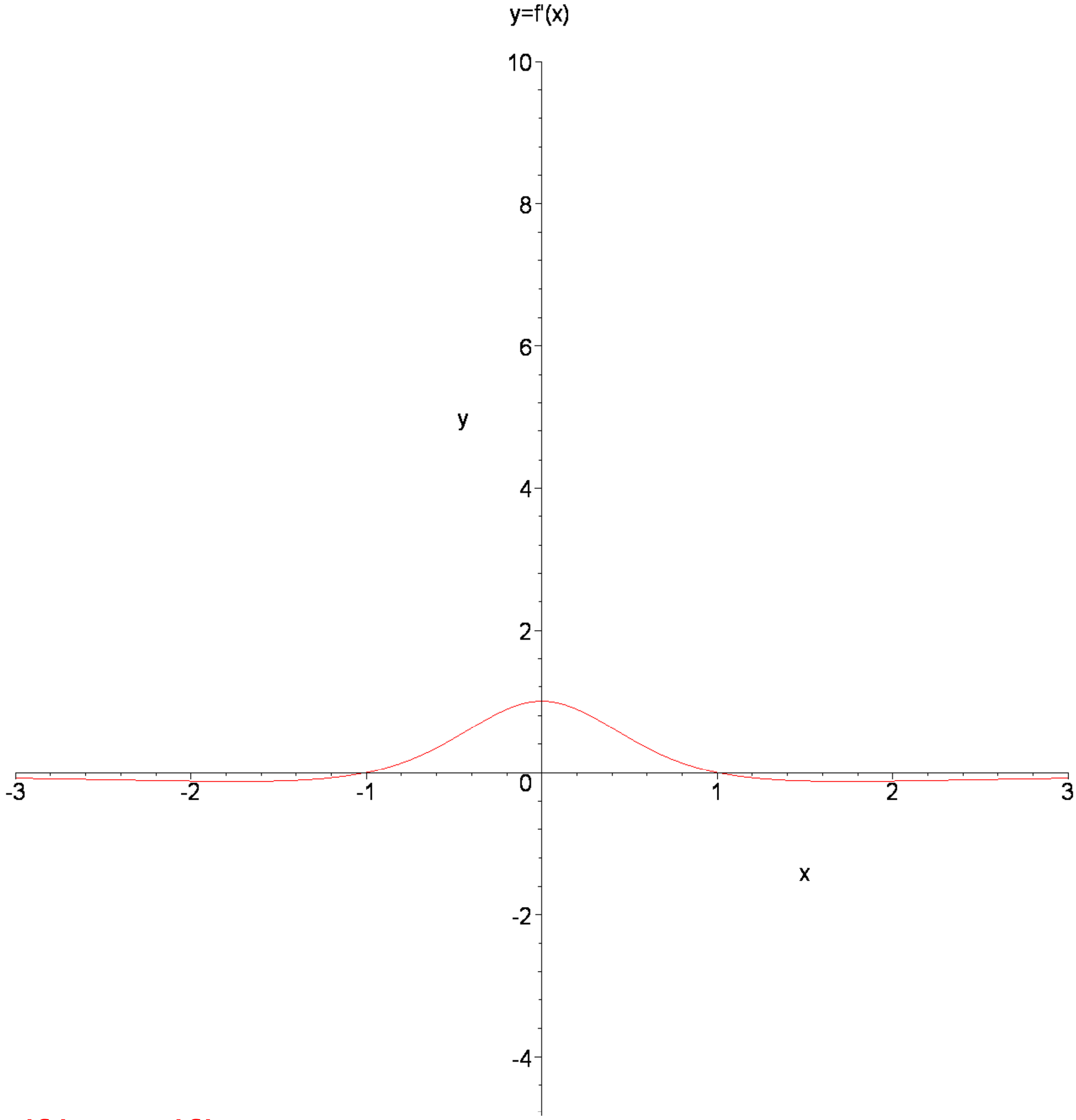
```
> f := x -> surd(x, 3)*surd((x-2)^2, 3);
```

$$f := x \to \text{surd}(x,\,3)\;\text{surd}((x-2)^2,\,3)$$

```
> plot(f(x), x=-3..3, title=`y=f(x)`);
```




```
>
> plot(dfdx(x), x=-3..3, y=-5..10, title=`y=f'(x)`, discont=true,
  color=red);
```




```
> dfdx := D(f);
```

$$dfdx := x \rightarrow \frac{1}{3}\,\frac{\mathrm{surd}(x, 3)\,\mathrm{surd}((x-2)^2, 3)}{x} + \frac{1}{3}\,\frac{\mathrm{surd}(x, 3)\,\mathrm{surd}((x-2)^2, 3)\,(-4+2\,x)}{(x-2)^2}$$

```
> solve(dfdx(x)=0, x);
```

$$\frac{2}{3}$$

```
>
>
>
```