Funkce komplexni promenne

11.Dokazte, ze je funkce holomorfni v C a vypoctete jeji derivaci

$\mathbf{e}^z = (\mathbf{e}^x)(\cos(y) + i\sin(y))$

Reseni:

u(x,y)=$\Re(\mathbf{e}^z) = (\mathbf{e}^x)(\cos(y))$

v(x,y)=$\Im(\mathbf{e}^z) = (\mathbf{e}^x)(i\sin(y))$

Obe tyto funkce maji spojite parcialni derivace vsech radu podle vsech promennych x, y , maji tedy totalni diferencial. Postaci tedy dokazat splneni Cauchy-Riemannovych podminek:

```
> realf:=(x,y)->(exp(x)*cos(y));
```

$$realf := (x, y) \to \mathbf{e}^x \cos(y)$$

```
> imf:=(x,y)->exp(x)*I*sin(y);
```

$$imf := (x, y) \to \mathbf{e}^x \sin(y)\, I$$

```
> r1:=diff(realf(x,y),x);
```

$$r1 := \mathbf{e}^x \cos(y)$$

```
> im1:=diff(imf(x,y),y);
```

$$im1 := \mathbf{e}^x \cos(y)\, I$$

```
> r2:=diff(realf(x,y),y);
```

$$r2 := -\mathbf{e}^x \sin(y)$$

```
> im2:=diff(imf(x,y),x);
```

$$im2 := \mathbf{e}^x \sin(y)\, I$$

Je jasne videt, ze r1=im1 a r2=-im2, takze C.-R. podminky jsou splneny.

Derivace je zrejma:

```
> diff(exp(z),z);
```

$$\mathbf{e}^z$$

```
>
```

```
>
```