Fourierovy rady

14. Rozvinte ve Fourierovu radu a napiste jeji Parsevalovu rovnost
f(x)=sign(x) na intervalu (-Pi,Pi)

```
> funkce:=x->signum(x);
```

$$funkce := x \to \mathrm{signum}(x)$$

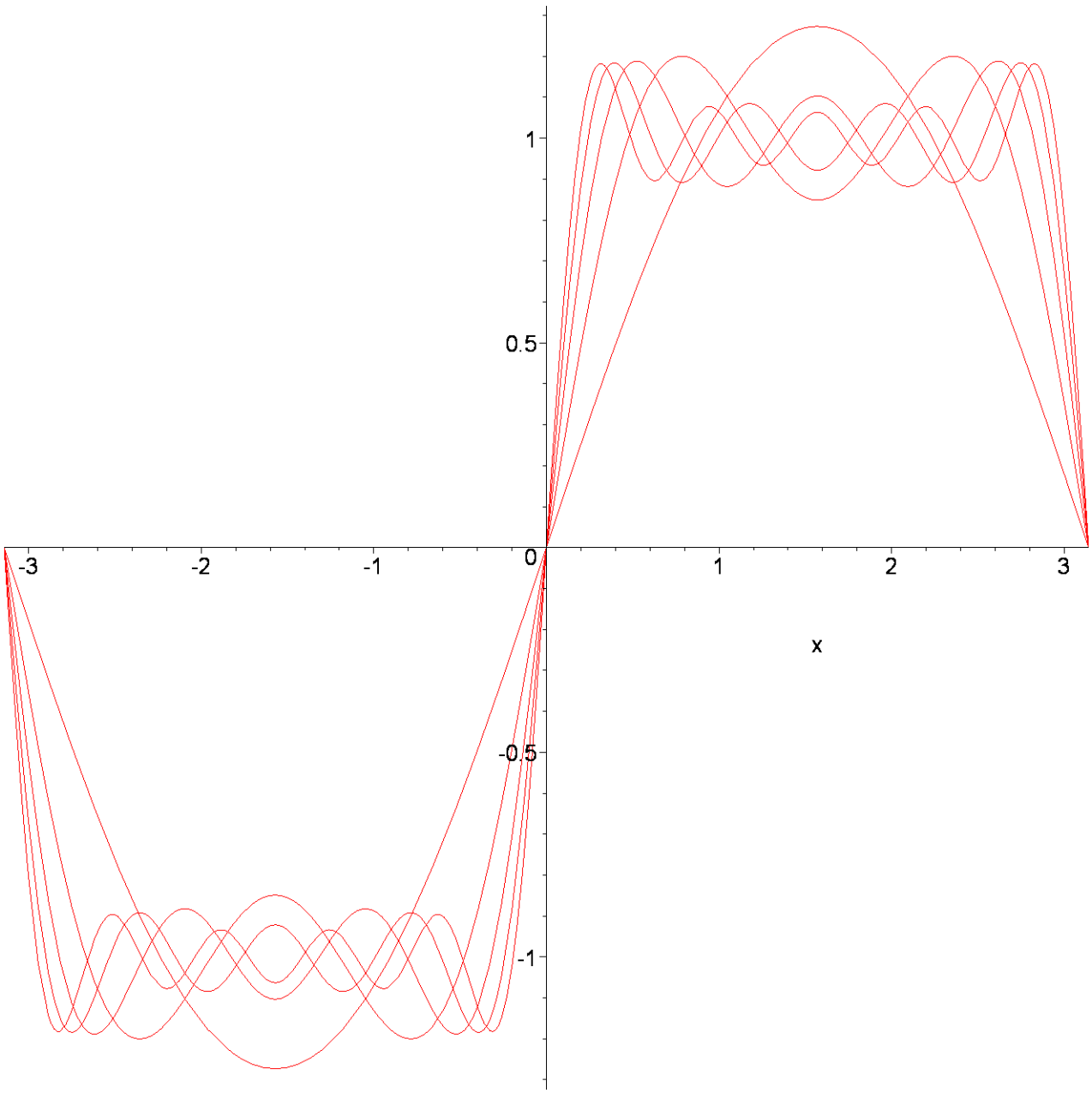
```
> a:=n->int(-cos(n*x),x=-Pi..0)/Pi+int(cos(n*x),x=0..Pi)/Pi;
```

$$a := n \to \frac{1}{\pi}\int_{-\pi}^{0} -\cos(n\,x)\,dx + \frac{1}{\pi}\int_{0}^{\pi} \cos(n\,x)\,dx$$

```
> b:=n->int(-sin(n*x),x=-Pi..0)/Pi+int(sin(n*x),x=0..Pi)/Pi;
```

$$b := n \to \frac{1}{\pi}\int_{-\pi}^{0} -\sin(n\,x)\,dx + \frac{1}{\pi}\int_{0}^{\pi} \sin(n\,x)\,dx$$

```
> f:= plot(funk(x),x=-3..3,discont=true,color=blue):
> fur:=seq(plot(a(0)/2 +sum(a(n)*cos(n*x)
  +b(n)*sin(n*x),n=1..m),x=-Pi .. Pi),m=0..10):
> display(fur,insequence=true):
> display(f,fur);
```




```
> m:=20:
> a(0)/2+sum(a(n)*cos(n*x)+ b(n)*sin(n*x),n=1..m);
```

$$\frac{4\sin(x)}{\pi}+\frac{4}{3}\frac{\sin(3x)}{\pi}+\frac{4}{5}\frac{\sin(5x)}{\pi}+\frac{4}{7}\frac{\sin(7x)}{\pi}+\frac{4}{9}\frac{\sin(9x)}{\pi}+\frac{4}{11}\frac{\sin(11x)}{\pi}+\frac{4}{13}\frac{\sin(13x)}{\pi}$$
$$+\frac{4}{15}\frac{\sin(15x)}{\pi}+\frac{4}{17}\frac{\sin(17x)}{\pi}+\frac{4}{19}\frac{\sin(19x)}{\pi}$$

Je zrejme, ze Fourierova rada pro funkci f(x)=sign(x) ma tvar

$$\frac{4\left(\sum_{k=1}^{\infty}\frac{\sin((2k-1)x)}{2k-1}\right)}{\pi}$$

Nyni urcime Parsevalovu rovnost:

```
> int(funkce(x)^2,x=-Pi..Pi)/Pi=(a(0)^2)/2+sum(a(n)*a(n)
```

```
   +b(n)*b(n),n=1..infinity);
```

$$2 = 2$$

Prepsano do hezciho tvaru

$$\frac{\pi^2}{2} = \sum_{n=1}^{\infty} \frac{(-1+\cos(n\,\pi))^2}{n^2}$$

```
>
```